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Rotated Branes and $N = 1$ Duality

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We consider configurations of rotated NS-branes leading to a family of four-dimensional $N = 1$ super-QCD theories, interpolating between four-dimensional analogues of the Hanany-Witten vacua, and the Elitzur-Giveon-Kutasov configuration for $N = 1$ duality. The rotation angle is the $N = 2$ breaking parameter, the mass of the adjoint scalar in the $N = 2$ vector multiplet. We add some comments on the relevance of these configurations as possible stringy proofs of $N = 1$ duality.

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1. Introduction

Recently, very explicit string realizations of Seiberg's $N = 1$ duality [1] have been proposed in a number of papers. They involve aspects of D-brane dynamics in non-trivial compactification manifolds [2], combined with standard T-duality, or more complicated structures in flat space including both D-branes and NS-branes [3]. A recent work with a unified view is [4].

We study some aspects of the configurations presented by Elitzur, Giveon and Kutasov (EGK) in [3], which describe a continuous family of type-IIA brane configurations interpolating between two Seiberg dual pairs in the simplest case. These manipulations rely heavily on non-trivial effects of brane dynamics described by Hanany and Witten (HW) in [5]. In this note, we exhibit a family of rotated brane configurations interpolating between a type-IIA four-dimensional analogue of the HW configurations, and the EGK configuration. This family of configurations with four-dimensional $N = 1$ supersymmetry is a microscopic model for the simplest deformation of four-dimensional $N = 2$ QCD into $N = 1$ QCD, by giving an $N = 1$ preserving mass to the adjoint chiral superfield in the $N = 2$ vector multiplet. In this way, we make contact with previous work of Argyres, Plesser and Seiberg in ref. [6].

2. Interpolating between the HW and EGK Configurations

We will consider the basic set-up of ref. [3] in type-IIA string theory: a configuration containing an NS_5 five-brane localized in the (x^6, x^7, x^8, x^9) directions, a second NS'_5 five-brane localized in (x^4, x^5, x^6, x^7), at the same value of x^7 as the NS_5 five-brane, and separated by an interval L_6 in the x^6 direction. We also have a Dirichlet four-brane D_4 with world-volume along $(x^0, x^1, x^2, x^3, x^6)$, stretched in the x^6 direction between the NS_5 and NS'_5 five-branes. Finally, we have a Dirichlet six-brane D_6 localized in (x^4, x^5, x^6). If we arrange N_c coincident four-branes and N_f six-branes, the previous configuration defines an $N = 1$ super-QCD with gauge group $U(N_c)$ and N_f flavours of quarks in the fundamental representation, along the four non-compact dimensions of the D_4 world-volume: the space (x^0, x^1, x^2, x^3).

The amount of supersymmetry is easily characterized. In terms of the ten-dimensional chiral and anti-chiral type-IIA spinors: $\varepsilon = \Gamma^0 \cdots \Gamma^9 \varepsilon$, $\overline{\varepsilon} = -\Gamma^0 \cdots \Gamma^9 \overline{\varepsilon}$, each NS-brane imposes the projections

$$\varepsilon = \Gamma_{NS}\, \varepsilon \quad , \qquad \overline{\varepsilon} = \Gamma_{NS}\, \overline{\varepsilon}, \tag{2.1}$$

where Γ_{NS} is the product of Dirac matrices along the brane world-volume directions. On the other hand, D-branes relate both ten-dimensional spinors by the constraint

$$\overline{\varepsilon} = \Gamma_D\, \varepsilon. \tag{2.2}$$

In the above configuration, the first five-brane NS_5 preserves 1/2 of the original ten-dimensional $N = 2$ supersymmetry. The second NS_5' breaks 1/2 of the remaining supersymmetry, as does the D_6 brane. These conditions leave four real charges or $N = 1$ in four dimensions. The NS_5 and D_6 conditions imply the relation

$$\overline{\varepsilon} = \Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^6\, \varepsilon, \tag{2.3}$$

so that another D_4 brane is allowed, extended in the $(x^0, x^1, x^2, x^3, x^6)$ directions, without any further breaking of supersymmetry. From the geometry of the configuration this means that the D_4 must stretch between the NS_5 and the NS_5', and be localized at the fixed common x^7 position.

It is easy to see that replacing the NS_5' by a second, displaced NS_5 leads to $N = 2$ supersymmetry on the non-compact part of the D_4 world-volume. This is a result of eq. (2.3) being a consequence of (2.1) and (2.2) for the NS_5 and D_6 branes. If we take the five-branes as rigid static objects for the purposes of defining the effective physics on the D_4 world-volume, the extra scalars in the adjoint representation required by $N = 2$ supersymmetry appear because the D_4 is now free to fluctuate in the (x^4, x^5) plane. So, we have $N = 2$ super-QCD with N_c colours and N_f flavours in a four-dimensional type-IIA generalization of the Hanany-Witten configurations[1].

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[1] The four-dimensional configurations follow from the ones considered in [5] by a T-duality in the x^4 direction, under the assumption that the NS-branes are inert under this transformation. Considered as a closed string background, the string metric component of the type-IIA five-brane has $g_{44} = 1$ when the x^4 dimension belongs to the world-volume. Therefore, it is unchanged by T-duality $g_{44} \to 1/g_{44}$, and we end up with a family of type-IIB configurations as in ref. [5], averaged over the compact x^4-circle.

This situation immediately suggests an interpolation between both types of configurations, by simply rotating the second NS_5 into the (x^8, x^9) plane, to define an NS'_5 brane. Such a rotation can be performed without breaking all the supersymmetries, according to the results of ref. [7]. The condition is that it can be written as an $SU(n)$ rotation for an appropriate complexification of space.

Define the complex planes $z = x^4 + ix^8$, $w = x^5 + ix^9$. Then, the NS_5 is stretched in the plane $\mathrm{Im}\, z = \mathrm{Im}\, w = 0$, whereas the final NS'_5 configuration lies on $\mathrm{Re}\, z = \mathrm{Re}\, w = 0$. Clearly, the rotation

$$z \to e^{i\theta} z \quad , \quad w \to e^{-i\theta} w \tag{2.4}$$

is in $SU(2)$ and leaves some unbroken supersymmetry. Since the starting configuration has four-dimensional $N = 2$ supersymmetry, and the final one at $\theta = \pi/2$ has $N = 1$, the minimal amount in four dimensions, we know that all rotated branes NS_5^θ leave exactly $N = 1$ supersymmetry on the D_4 world-volume. We can see this more explicitly by using (2.1)–(2.3). Defining

$$a_z = \frac{1}{2}\,(\Gamma^4 + i\Gamma^8) \quad , \quad a_w = \frac{1}{2}\,(\Gamma^5 + i\Gamma^9), \tag{2.5}$$

the condition for unbroken supersymmetry at an angle θ becomes

$$(a_z + a_z^\dagger)(a_w + a_w^\dagger)\,\varepsilon = (e^{i\theta} a_z + e^{-i\theta} a_z^\dagger)(e^{-i\theta} a_w + e^{i\theta} a_w^\dagger)\,\varepsilon, \tag{2.6}$$

and both the vacuum $|0\rangle$ and the top state $a_z^\dagger a_w^\dagger |0\rangle$ of the system of two oscillators survive. Moreover, they have the same ten-dimensional chirality. We can take any of the two states to build spinors out to the rest of Dirac matrices. We have six extra Dirac matrices which give a total of 2^3 states. These are reduced by a factor of $1/4$ by the NS_5 and D_6 conditions, leaving two states on top of each of the z–w vacua. In all, we have four states, corresponding to $N = 1$ in four dimensions.

The starting configuration at $\theta = 0$ with $N = 2$ supersymmetry contains an adjoint scalar coming from fluctuations of the D_4 in the (x^4, x^5) plane, $\Phi = X^4 - iX^5$, where $X^{4,5}$ represent the $N_c \times N_c$ D-brane position matrices. On the other hand, the final EGK configuration at $\theta = \pi/2$ has no scalar moduli, under the assumption of rigidity of the background branes. Therefore, it is natural to interpret the rotation angle θ as a mass parameter for the $N = 2$ adjoint field, inducing a superpotential of the form $W_\mu = \mu\, \mathrm{Tr}\Phi^2$. For a more precise statement we need some discussion on the rigidity of the background branes.

3. Brane Angles and $N = 2$ Breaking Mass

A basic assumption of the constructions in [5] and [3] is the rigidity of the background branes. In other words, one never considers scalar moduli corresponding to D_4 fluctuations in the transverse directions common to both the D_4 and the background branes. We can characterize this rigidity at a quantitative level by adding convenient mass terms for the corresponding scalar fields. For example, in the $\theta = 0$ configuration, we would "freeze" the transverse fluctuations in the (x^8, x^9) plane $\Phi' = X^8 + iX^9$, by giving them a large mass μ_0, at the D_4 end-points attached to the NS_5 branes. The full five-dimensional action on the D_4 world-volume takes the form

$$\begin{aligned} S_{5d} = \int d^4x\, dx^6\, \mathcal{L}_{\rm bulk} + \mu_0 \int_{x^6=0} d^4x\, d^2\theta\, \mathrm{Tr}\left(\Phi'(x^6=0)\right)^2 + \\ +\mu_0 \int_{x^6=L_6} d^4x\, d^2\theta\, \mathrm{Tr}\left(\Phi'(x^6=L_6)\right)^2 \;\; + \mathrm{h.c.} \end{aligned} \tag{3.1}$$

After dimensional reduction at small L_6 we just keep zero modes in the x^6 direction and then $\Phi'(x^6 = 0) = \Phi'(x^6 = L_6)$. We end up with

$$S_{4d} = L_6 \int d^4x\, \mathcal{L}_{\rm bulk} + 2\mu_0 \int d^4x\, d^2\theta\, \mathrm{Tr}(\Phi')^2 \;\; + \mathrm{h.c.} \tag{3.2}$$

In the decoupling limit $\mu_0 \to \infty$, the Φ' fields are frozen[2], and we are left with the $N = 2$ four-dimensional theory, with bare gauge coupling $g_{\rm bare} \sim L_6^{-1/2}$.

It is now very easy to incorporate the rotation of the second NS_5. We simply modify the boundary action at $x^6 = L_6$, by writing a superpotential

$$W_\theta(x^6 = L_6) = \mu_0\, \mathrm{Tr}(\Phi'_\theta)^2, \tag{3.3}$$

with $\Phi'_\theta = X^8_\theta + iX^9_\theta$, and

$$\begin{aligned} X^8_\theta =& X^4 \sin\theta + X^8 \cos\theta \\ X^9_\theta =& X^9 \cos\theta - X^5 \sin\theta. \end{aligned} \tag{3.4}$$

Working out the dimensional reduction we find the following superpotential in four dimensions:

$$W_\theta = \mu_0\, (1 + \cos^2\theta)\, \mathrm{Tr}(\Phi')^2 + \mu_0 \sin^2\theta\, \mathrm{Tr}(\Phi)^2 + \mu_0 \sin 2\theta\, \mathrm{Tr}\,(\Phi\, \Phi'). \tag{3.5}$$

[2] A dynamical motivation for the rigidity of the NS branes as compared to the D_4 branes could be found in the parametrically larger tension, at weak coupling $T_{NS} \sim g_{\rm st}^{-2} \gg g_{\rm st}^{-1} \sim T_D$.

Thus, after diagonalization for small θ, there is a heavy field with mass of order μ_0, and a light field with mass parameter

$$\mu = \mu_0 \frac{\sin^2\theta}{1+\cos^2\theta} \sim \frac{\mu_0}{2}\theta^2. \tag{3.6}$$

For $\theta \sim \pi/2$ the two fields are decoupled, as corresponds to the absence of moduli in the EGK configuration.

The "duality trajectory" of brane configurations described in [3] is easily generalized to the rotated configurations, as the corresponding intermediate Higgs phases with a non-zero Fayet-Iliopoulos coupling (FI) do exist for the deformed $N=2$ theories. Indeed, for $\theta = 0$, the EGK trajectory realizes explicitly an "$N=2$ duality" between $U(N_c)$ and $U(N_f - N_c)$ theories, similar to the one described in [2], [4], [8].

The final configuration obtained after passing through the Higgs branch with a non-zero FI term consists of $N_f - N_c$ D_4 branes stretched in the x^6 between two parallel NS_5 branes, together with N_f D_4 branes stretched between the second NS_5 and N_f D_6 branes[3]. The fundamenal type-IIA strings stretching between D_4 branes on both sides of the second NS_5 provide the N_f massless quark flavours. Notice that there are no extra "magnetic mesons" in this $N=2$ configuration, since the N_f D_4 branes between the second NS_5 and the D_6 branes are rigid. This is, however, a subtle point, since one might argue that a flavour gauge group $U(N_f)$ should be present, with inverse squared coupling proportional to the x^6-distance between the D_6 branes and the NS_5 brane. Then, $N=2$ supersymmetry would imply the existence of an adjoint superfield for the flavour group with $N=2$ couplings, which would naturally qualify for Seiberg's magnetic mesons. This is a subtle question because of the very particular structure of D_4–D_6 linking (one to one). In any case, if we stick to the convention that background branes are rigid for the purposes of defining massless dynamics on the D_4 world-volume, there is apparently no room for flavour gauge group in the $N=2$ version of the final EGK configuration.

In ref. [4], a similar arrangement of branes was proposed, realizing an $N=1$ duality trajectory. The starting configuration is an NS_5' brane connected to an NS_5 brane by N_c D_4 branes stretched in the x^6 direction, which in turn is further connected by N_f D_4 branes to a second NS_5' brane. The duality trajectory proceeds by switching the

[3] This is not an s-configuration, in the terminology of ref. [5], because the local linking of D_4 branes to D_6 branes is one to one.

positions of the first NS_5' and the middle NS_5; the change of gauge group comes about by reconnection of branes at the middle background brane. In this construction, there is an obvious flavour–colour symmetry from the geometry of the configurations, and we are clearly describing $U(N_c) \times U(N_f)$ or $U(N_f - N_c) \times U(N_f)$ gauge theory with matter in the $(N_c, N_f) + \text{h.c.}$. In this context, regarding $U(N_f)$ as a global flavour group is more a question of making its gluous very weakly coupled by adjusting the brane distances. The rotated configurations considered in this paper can be trivally extended to this case: by a complex rotation of the middle NS_5 into the (x^8, x^9) directions we achieve an $N = 2$ configuration with three parallel NS_5' branes connected by D_4 branes as above. Here we *do* find the appropriate adjoint scalars required by $N = 2$ supersymmetry both in the "colour" and "flavour" sectors, because both sets of D_4 branes are free to fluctuate in the (x^8, x^9) directions.

Coming back to the EGK configurations, the effect of turning on a rotation angle of the middle NS_5 is again a soft $N = 1$ mass $\mu \sim \theta^2$ for the adjoint $\Phi = X^4 - iX^5$. At the same time, the inverse effect occurs as $\theta \to \pi/2$, near the EGK configuration. Now the same analysis applied to the N_f "flavour" D_4 branes leads to a mass $\mu_f \sim (\theta - \pi/2)^2$ for the "flavour adjoint" $\Phi'_f = X^8_f + iX^9_f$, where $X^{8,9}_f$ denote the corresponding position matrices of the N_f D_4 branes stretching from NS_5' and the N_f D_6 branes. These are just Seiberg's extra magnetic mesons coming down as $\theta \to \pi/2$.

Thus, the complete picture in the $(g_{\text{bare}} \sim L_6^{-1/2}, \mu)$ plane is strongly reminiscent of similar discussions in the field theoretical context of ref. [6].

4. Concluding remarks

In this note we have shown that $N = 2$ and $N = 1$ brane configurations, appropriate for discussions of four-dimensional duality, can be connected by a rotation process of one of the branes. This realizes the simplest deformation on $N = 2$ QCD by the lifting of the adjoint chiral superfield. It would be very interesting to sharpen the analogy between this two-parameter family of theories and the analogous treatment in ref. [6]. These authors pinned the degrees of freedom of the magnetic dual by slightly breaking $N = 2$ to $N = 1$ with $\mu \ll \Lambda_{N=2}$, the key point being that vacua at the roots of the Higgs branches with the right properties are not lifted. Then, deforming the theory by increasing μ past $\Lambda_{N=2}$, one finds the microscopic electric description. This process is clearly analogous to

the interplay between brane motion (variation of the bare couplings), and angle rotation ($N = 1$ breaking mass).

There are some superficial differences, though. For example, in the field theoretical treatment of [6], the dual quarks and gluons and the magnetic mesons evolve from vacua at the baryonic and non-baryonic roots respectively. The distance between these roots is of order $\Lambda_{N=2}$, the strong interaction scale, which vanishes in the decoupling limit $\mu \to \infty$ with $\Lambda_{N=1}^{3N_c - N_f} = \mu^{N_c} \Lambda_{N=2}^{2N_c - N_f}$ fixed, thereby merging into a single vacuum. In the brane treatment, however, the magnetic mesons already appear at a microscopic level. In this sense, it is interesting to note that they are absent for the $N = 2$ configuration, perhaps in analogy with the previously mentioned low-energy splitting between baryonic and non-baryonic roots.

A more explicit connection with ref. [6] might be achieved by regarding the bare coupling of the effective four-dimensional theory fixed at the string scale $g^2_{\rm bare} \sim g_{\rm st}$, and considering the physics at a scale M, with $M\, L_6 \ll 1$ kept fixed as we move L_6. Then, brane motion really corresponds to renormalization group flow, by changing the scale M. Taking M to the infrared, and at the same time deforming the theory to $\mu \to \infty$ at different relative velocities, identifies both Seiberg duals at intermediate scales. The full field theoretical analysis is recovered in the complete decoupling limit for infinitely rigid branes; according to (3.6), we take $\mu_0 \to \infty$ and $\theta \to 0$, keeping μ fixed. However, in this way we lose the $\theta \sim \pi/2$ region and the stringy characterization of the magnetic mesons. So, it might be useful to keep a finite μ_0 after all.

The requirement of having to pass through the Higgs branch, in order to avoid an infinite coupling singularity in the stringy setting, could be related to the fact that, as we reduce L_6 and take M past $\Lambda_{N=2}$ into the infrared, the baryonic Higgs cone splits from the classical unbroken $SU(N_c)$ vacuum. Therefore, we need to go through the Higgs branch in order to reach the infrared-free $SU(N_f - N_c)$ vacuum, unless we take the "short-cut" through the Coulomb phase, which corresponds in the brane language to the relative splitting of the N_c D_4 branes in the (x^4, x^5) directions. In this respect, a disturbing feature of the brane configurations is the fact that one always gets unitary gauge groups, rather than special unitary gauge groups. As pointed out in [5], this means that baryon number is effectively gauged, and the baryonic branch is not present in the $U(N_c)$ moduli spaces. In order to fully compare the brane approach to the discussion in [6], one should somehow restore the baryonic Higgs branch in the brane picture.

These analogies should become more specific. This is an important point in elucidating the $N = 1$ duality mapping, because the continuous family of brane configurations interpolating between dual pairs does not guarantee the infrared equivalence of the two theories [4]. Indeed, they are clearly inequivalent along the $\theta = 0$, $N = 2$ slice, which connects an asymptotically free theory with an infrared-free theory for $N_c \leq N_f \leq 2N_c$. Therefore, it is unlikely that purely microscopic considerations will qualify for an unambiguous proof of $N = 1$ duality, and some low-energy input, in the spirit of [6], will be necessary.

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